Qwirkle: From fluid reasoning to visual search.

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Abstract

This study focuses on subjects' performance and strategies in the board game called Qwirkle. Earlier scientific studies mostly used Qwirkle as a tool for improving mathematical reasoning skills in children. However, Qwirkle may additionally require significant visuo-spatial processing. Results from our study indicate that subjects use a simple local maximum strategy in which scores at local decision points of the game are maximized. As was expected, the strategy requires mathematical reasoning skills. However, results also suggest that subjects' performance is significantly affected by visual search skills. We conclude that the visual and reasoning systems are deeply intertwined. On the one hand, the reasoning outcome is highly dependent on both attentive and pre-attentive visual knowledge. On the other hand, visual processing requires capability of reasoning on concepts more higher level than visual features.

Introduction

In our previous studies, we have tackled two major issues that commonly occur in complex problem solving tasks. The first is the role of our visual system as a major source of real-time information (Nyamsuren & Taatgen, 2013a, 2013b, 2013c). The result of this research was the Pre-Attentive and Attentive Vision module (Nyamsuren & Taatgen, 2013a). The second is task-general declarative and procedural knowledge that enables us to reason and understand how specific problems should be approached and solved. The result of this study was the Human Reasoning Module (Nyamsuren & Taatgen, 2013d).

Isolated understanding of human reasoning and human vision is not enough to understand the cognitive underpinning of human problem solving. For a complete picture, we should also understand how reasoning and visual systems interplay in a single coherent cognitive architecture. We have yet to draw a bridge between PAAV and HRM that allows full interaction between the two modules. Our earlier model (Nyamsuren & Taatgen, 2013a) of spatial reasoning task (Byrne & Johnson-Laird, 1989) only scratched the surface of necessary cognitive functionalities connecting the two systems. Therefore, additional investigation based on a more complex task is required.

Qwirkle

We have chosen the board game of Qwirkle[1] as a representative of a problem solving task that requires both complex visual processing and reasoning. Qwirkle is a competitive game that requires at least two and at most four players. It is a game based on tiles. Each tile has a shape of a certain color. There are six unique colors and six unique shapes resulting in 36 unique tiles shown in Figure 1. In total, there are 108 tiles with three copies of each unique tile. Tiles are usually kept in a bag so they are not visible to players.

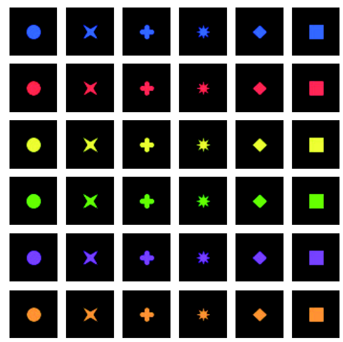

Figure 1: 36 unique tiles of Qwirkle.

The game starts with each player drawing six random tiles from the bag. Next, another three tiles are drawn randomly and put on the center of the board face up next to each other. None of the players can see the other players' tiles. Players make moves in turns. During her turn, a player can perform either one of two actions: put one or more tiles with the same color or shape on the board, or replace one or more tiles from her stack with random tiles from the bag. The replaced tiles are put back into a bag. After putting tiles on a board, a player replenishes her six-tile stack with new tiles randomly picked from a bag. It is not necessary for a player to put all tiles in the same row or column. Instead, individual tiles can be put in places where they fit best. There are two main rules governing where a tile can be put. First, a tile should be put next to another tile that is already on the board. Second, any sequence of tiles on the board should have either the same color and different shapes or different colors and the same shape. The longest possible sequence consists of six tiles and is referred to as a *qwirkle*.

A player receives points for each tile put on a board. A player who puts the final tile of a qwirkle receives 12 points (Figure 2a), the maximum amount of points possible to get from a single tile sequence. Otherwise, scoring is based on the length of the sequences a new tile forms. For example, forming a sequence with two tiles results in two points (Figure 2b). If a newly put tile forms a horizontal sequence

[1] Qwirkle is a game by MindWare (www.mindware.com).

with three tiles and a vertical sequence with four tiles then the move results in seven points (Figure 2c).

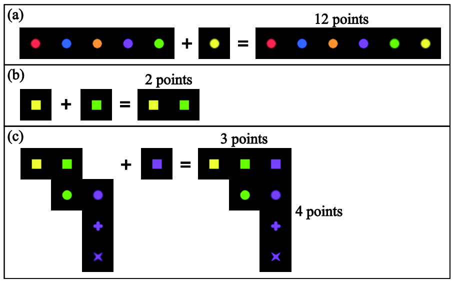

Figure 2: Scoring rules in Qwirkle.

The game ends if a player puts her last tile on a board and there are no more tiles in the bag to replenish from. A player who finishes the game first receives bonus six points. The player with the highest amount of points is the winner of the game. A single-player version[2] of the game can be found at www.ai.rug.nl/~n_egii/qwirkle/.

What makes Qwirkle interesting?

Qwirkle requires a significant degree of reasoning skills including mathematical problem-solving skills and the ability to consider alternative options. In fact, Qwirkle has been used to improve mathematical problem solving skills in schoolchildren (Klanderman, Moore, Maxwell, & Robbert, 2013; Maloy, Edwards, & Anderson, 2010). Furthermore, Mackey, Hill, Stone and Bunge (2011) argued that computerized and non-computerized reasoning games, with Qwirkle among them, improve children's fluid reasoning, the capacity to think logically and solve problems in novel situations (Cattell, 1987; Horn & Cattell, 1967). Although not widely recognized in previously mentioned studies, Qwirkle has a significant visual component in it. On the one hand, it requires visuo-spatial reasoning. In fact, Mackey et al. also found that subjects playing Qwirkle along with other games significantly improved their spatial working memory. On the other hand, Qwirkle requires basic processes of visual feature-based search. This dual nature of Qwirkle that involves both the reasoning and visual systems makes the game an ideal candidate for investigating how both processes work together in single task.

Objectives

The purpose of this study is to investigate the form of a strategy the players use in Qwirkle. Revealing the strategy is crucial for detailed understanding of the types of cognitive processes involved in the game. The overall strategy can be divided into individual steps, and each step can be assigned into specific cognitive resources. In this way, it is possible to investigate the specific roles the reasoning and visual systems play in Qwirkle. Furthermore, it should give an insight into the interplay between these two systems.

[2] The online and experiment versions of the Qwirkle game were developed and used under the terms of fair use for non-profit educational purpose only.

Experiment

Subjects

In total, 17 subjects participated in the experiment. Results from three subjects were excluded from the analysis due to technical errors and the high amount of noise in the eye tracking data. The average age of the subjects was 22 (*SD = 3.29*). There were six female and eight male subjects.

Design and procedure

Each subject was required to play ten games against a single computer opponent. The computer opponent had the simple strategy of maximizing its score for each turn. In each turn, the computer opponent would consider all possible unique combinations out of six (or less) tiles it has in its stack. For each combination of tiles, the computer opponent found a combination of positions on the board that resulted in maximum amount of points. Finally, the combination of tiles and corresponding combination of positions on the board that gave the maximum possible number of points for the turn were chosen as the computer's move for the turn. Computer opponent did not plan ahead or consider subject's moves. Hence, it is not the optimal strategy for the game. Subjects were not informed about the strategy used by the computer opponent.

The experiment was divided into two blocks of five games each: a hint block and a no-hint block. In the hint block, the subject received hints at the start of each of her turns. A hint consisted of one of the six tiles in the subject's stack being highlighted with a red frame. The hint indicated that the tile belongs to a combination of one or more tiles that results in the highest possible score for the turn. Subjects were given instructions about the meaning of the hint. Subjects were also explicitly told that they were free to ignore the hint and pursue their own strategy. Half of the subjects started the experiment with a hint block while the other half started with a no-hint block. Hints served as a good reference point to deduce the strategy if subjects chose to use it.

A single game can last for quite a long time, especially if there are only two players. Therefore, any single game used only 54 tiles instead of 108 tiles. All 54 tiles were chosen randomly for each game. In addition, subjects were limited to 90 seconds to make moves in each turn. Figure 3 shows the example screen capture of the game during an experiment. The game board had a size of 15×15 cells. All games started with subjects' making the first moves.

All subjects were requested to read the game instructions and play an online version of it prior coming to the experiment. The online version of Qwirkle was similar to the version used in the experiment except having no computer opponent. At the beginning of the experiment, an experimenter again explained the instructions to the subject. Subjects also had an opportunity to play two practice games:

one with and one without hint. Results from practice trials were not included in the analysis.

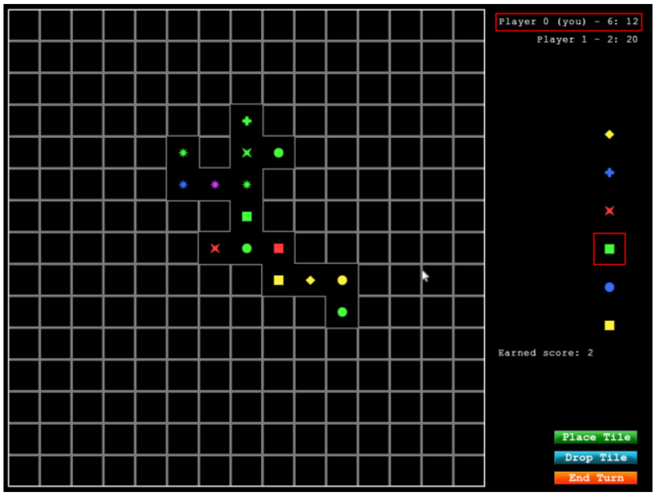

Figure 3: Example screen capture of Qwirkle game during an experiment.

Eye tracking

An EyeLink 1000 eye tracker was used for recording the eye movements. It is a desktop-mounted remote eye tracker with monocular sampling rate of 500 Hz and a spatial resolution of <0.01° RMS. The card images were shown on a 20-inch LCD monitor with screen size of 1,024×768 pixels and screen resolution of 64 pixels/inch. Subjects were asked to use a chin-rest to fix the head position during a recording. The tile image had a size of 50×50 pixels, or 1.62º×1.62º in angular size. The image of a shape within a tile fitted inside a square of 15×15 pixels, or 0.49º×0.49º in angular size. Angular sizes were calculated based on a viewing distance of 70 cm. The gaze position was calculated using the eye's corneal reflection captured using an infrared camera compensated for head movements. The eye tracker's default parameters were used to convert gaze positions into fixations and saccades. The calibration of an eye tracker was performed at the start and during the experiment, if necessary. A calibration accuracy of 0.8° was considered as an acceptable measure. Before each game, subjects were asked to do a drift correction as an additional corrective measure.

Action log and questionnaire

The progress of each game was recorded in a log file. The log file contained information about every action (placing a tile on the board or replacing a tile) performed by both the subject and the computer opponent. The log file contained sufficient information to restore any player's state or board state at any time during the game. At the end of the experiment, subjects were asked to fill in short questionnaire. Subjects were requested to provide information about their expertise level and previous experience with Qwirkle. They were also asked a few specific questions regarding the strategy such as preference toward any attribute, predicting opponent's moves or planning several turns ahead.

Experiment results

According to the questionnaires, none of the subjects had previous experience of playing Qwirkle prior to registering for the experiment. However, all subjects played the online version of the game prior coming to the experiment.

Attribute preference

In our previous studies with the card game of SET (Nyamsuren & Taatgen, 2013b), we have found that players had a preference for the color attribute over any other attribute such as shape. Surprisingly, we were not able to find any evidence of attribute preference in Qwirkle. We have tested whether subjects used combinations of tiles with the same color more than combinations of tiles with the same shape and vice versa. The usage did not significantly differ from one another. The statistical results also match with answers provided in the questionnaires. Eight subjects reported absence of preference toward either color or shape. Three subjects reported preference toward color, and another three subjects reported preference toward shape. In overall, there is no overall preference either toward color or shape.

Subjects' scores

Figure 4 shows subjects' performance in terms of proportions of games won and total amount of points gathered during an experiment. Subjects were sorted in increasing order of their total scores. Three subjects who won 50% of games each showed the highest performance. Two subjects who won only 10% of games each showed the lowest performance. There is a strong correlation between number of games won and total points gathered, $r(12) = 0.64$, $p = 0.014$. Increasing a score is not the only possible strategy in Qwirkle and does not necessarily guarantee a victory. For example, 43% of subjects reported in the questionnaire that they tried to block the opponent from completing the qwirkle. Similarly, 57% of subjects reported that they would be hesitant to put a fifth tile in a sequence without having a sixth tile (the opponent may put the final tile in the next turn). These strategies are highly situational, but still may affect the outcome of a game. Furthermore, a high score does not guarantee a victory, since it is always relative to the opponent's score. Nevertheless, the significant correlation suggests that gathering more points improves the chance of winning the game. Further in this work, we will treat the total score as a main indicator of subjects' performance.

If players were at least as good as the computer opponent then the success rate should be around chance probability of 50%. However, subjects had a relatively low success rate with on average three wins out of ten games. This result already indicates that subjects either used a strategy that is inferior to the one used by the computer opponent or used the same strategy but failed on some of the steps during the implementation. The second option is more likely considering that the computer opponent used a very simple strategy. It is not feasible to simplify the strategy even further unless subjects were making completely random moves.

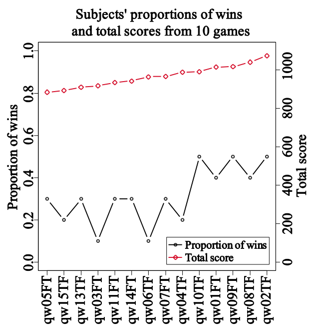

Figure 4: Proportions of wins and total number of points gathered by each subject during ten games.

One way repeated measures ANOVA applied to subjects' final scores shows no significant performance differences between the Hint and No-hint conditions $F(1, 13) < 1$. It is not surprising considering that scores gathered in individual games can differ significantly for the same subject. Analyses based on higher-granularity data described in following sessions show that there is indeed a difference between two conditions.

Figure 5 shows the mean scores gathered by subjects during individual turns in the Hint and No-hint conditions. In the figure, subjects were again sorted in increasing order of their total scores. The figure shows that there is a significant difference between subjects with low- and high-performance in No-hint condition. High performance subjects were able to gather at least two more points per turn than low performance subjects. Subjects' mean turn scores in No-hint condition significantly correlate with subjects' total scores, $r(12) = 0.87$, $p < 0.001$.

Next, Figure 5 shows that providing a hint helped the subjects to increase turn scores. As a result, the difference between low and high performance subjects is less prominent in Hint condition than in No-hint condition. It is supported by insignificant correlation between subjects' mean turn scores in Hint condition and total scores, $r(12) = 0.47$, $p = 0.1$. This result suggests that both low- and high performance subjects may have been using the same strategy. Low performance subjects may have been more prone to making mistakes while implementing the strategy. However, providing hints may have helped them to decreases chances of mistakes. There is no significant effect of Hint and No-hint order on mean turn scores. Neither, there is a significant effect of the order on subjects' average trial score.

Finally, given opportunities such as hints, subjects chose to maximize their turn scores. This fact suggests that subjects may have been using the same strategy as the computer opponent: maximize points gathered in each turn. The next section investigates further to confirm this assumption.

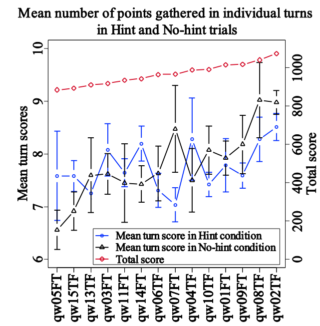

Figure 5: Mean number of points gathered by each subject in an individual turn in Hint and No-hint conditions. Subjects were sorted in ascending order of their total points.

Subjects' moves

Previously, we have suggested that subjects used the same strategy as the computer opponent. The core of the strategy is to find an optimal combination of moves that results in the highest possible score for the turn. It is essentially a local maximum strategy because it tries to maximize local reward at individual turns rather than a global reward from a sequence of turns. If subjects indeed used the local maximum strategy then proportions of turns with the highest possible scores obtained should increase as subjects' total scores increase. Secondly, the same proportions should be higher in the Hint condition than in the No-hint condition. Those proportions were calculated for each subject and separately for Hint and No-hint conditions. The results are shown in Figure 6a. The data on the figure confirms that both previous assumptions are true.

Firstly, subjects with higher total scores were more successful at getting highest possible turn scores. However, correlation between subjects' proportions in Hint condition and total scores is not significant, $r(12) = 0.53$, $p = 0.053$. On the other hand, there is a significant correlation between proportions in No-hint condition and total scores, $r(12) = 0.74$, $p = 0.003$. Results of correlation tests again suggest that low and high performance subjects differ mostly in No-hint condition while presence of the hint helps to negate skill differences among subjects.

Secondly, proportions are generally higher in the Hint condition than in the No-hint condition. On average, subjects were able to find the optimal combinations in 56% (SE = 3%) of the turns in Hint condition compared to 45% (SE = 3%) in No-hint condition. This difference is significant according to one-way repeated measures ANOVA, $F(1, 13) = 11.28$, $p = 0.005$.

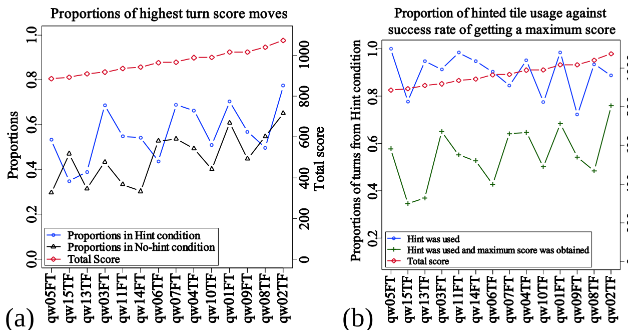

Figure 6: (a) Proportions of turns where subjects got highest possible scores. (b) Proportions of turns where subjects used tiles provided as hints in their moves and proportions of turns where subjects were able to get maximum scores while using the hinted tile.

Because subjects were explicitly told that they can ignore hints, it is possible that subjects chose to do so most of the times. This could explain why subjects have relatively low success rate even in Hint condition. Figure 6b shows how often subjects used the hinted tile. The figure shows that subjects have chosen to use the hinted tile, on average, in 90% (*SE* = 2%) of the turns where hint was provided. However, the figure also shows that the success rate of obtaining the maximum turn score is still much lower (*M* = 55%, *SE* = 3%) even when the hint was used. It is likely that subjects often failed during one of the two steps described previously. The test of correlation between total scores and proportions of hint usage is not significant, $r(12) = -0.18$, $p = 0.54$. The insignificant correlation test indicates that usage of a hint by itself does not guarantee success in the getting the maximum turn score.

There is again no effect the order of blocks on subjects' performance (Figure 7). Firstly, there is an overall learning effect from the first block to the second block. The learning effect is independent of order of two conditions. Secondly, subjects perform better at getting the highest score when a hint is given. The positive effect of the hint is quite significant. For example, the group of subjects who started with a Hint condition show lower performance in the second block with No-hint condition. Even the learning effect is not enough to compensate for the absence of a hint.

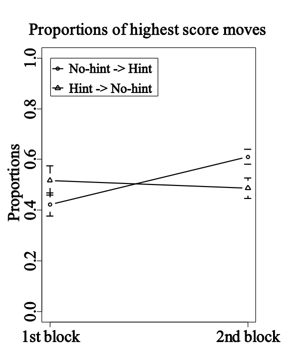

Figure 7: Effects of block order and trial type on proportions of turns where subjects got highest possible scores.

The local maximum strategy

The analyses from the last two sections support our hypothesis that subjects used the strategy of maximizing their turn scores. It is a simple strategy that can be described in two steps:

1. Identify an optimal combination of tiles that is likely to result in the highest possible score.
2. Identify an optimal combination of board positions for a chosen combination of tiles that is likely to result in the highest possible score.

Despite the simplicity, subjects were prone to making mistakes that prevented them from getting the maximum score as suggested by the low success rate in Figure 6a. Figure 6b leads us to conclude that, even given a hint, subjects still may fail to find a proper combination of tiles or a proper combination of board positions to put those tiles.

Optimal combination of tiles To find out how often subjects failed during the first step, we have calculated the proportions of turns where subjects used a combination of tiles that could have resulted in the maximum possible points for that turn (it does not necessarily mean that subjects actually got maximum points).

Figure 8a shows that subjects were extremely good at finding a combination of tiles that could have resulted in the highest possible score for the turn. Whenever subjects used the hinted tiles, they were able to find the proper combination of tiles in 90% (*SE* = 2%) of the turns on average (the blue line in Figure 8a). Furthermore, even if no hint was provided or subjects chose to ignore the hint, subjects were able to find alternative combo that could have resulted in the highest possible score in 75% (*SE* = 2%) of all turns (the black line in Figure 8a).

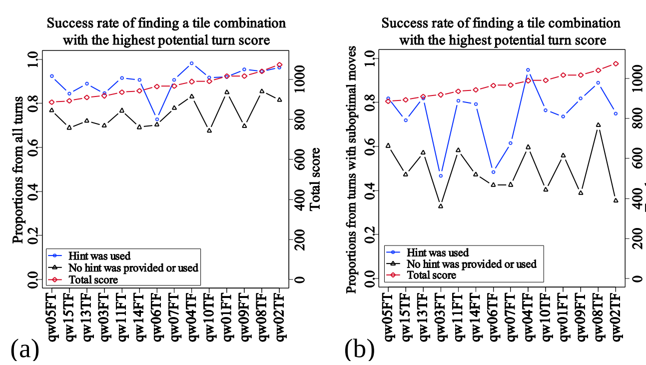

Figure 8: Proportions of turns where subjects were able to find the combinations of tiles that could have resulted in the highest possible turn scores. The proportions were calculated from (a) all turns and (b) only from those turns where subjects failed to get the maximum possible score.

If the same proportions are calculated using only those turns where subjects failed to get maximum possible score then the proportions are still quite high (Figure 8b). The average proportions are 74% (*SE* = 4%) and 49% (*SE* = 3%) in turns where hint was used and turns where hint was ignored or not provided at all.

None of the proportions shown in Figure 8 significantly correlated with subjects' total scores. It suggests that failures in the step 1 of the strategy cannot fully account for the performance differences among subjects. Furthermore, Figure 8b suggests that at least 49% - 74% of all failures to get maximum possible turn score should be due to the failure in the second steps of the strategy. Subjects do pick the right combinations of tiles, but not necessarily put them on the optimal spots on the board.

Optimal combination of board positions Previous analyses suggest that an important process defining subject's performance is how well she can find an optimal combination of board position that maximizes the amount of gathered points. If this proposition is true then a subject with a lower total score should fail more than a subject with a higher total score during step 2 of the strategy. It can be easily tested by calculating proportions of turns where subjects were able to find optimal board positions for combination of tiles they have chosen in each turn. Figure 9 shows those proportions calculated for each subject. On average, subjects succeeded in finding an optimal combination of board positions in 67% (SE = 3%) of turns. The proportions are strongly correlated with subjects' total scores, r(12) = 0.65, p = 0.01. The significant correlation suggests that the ability to find an optimal combination of board positions is a strong indicative of subjects' performances.

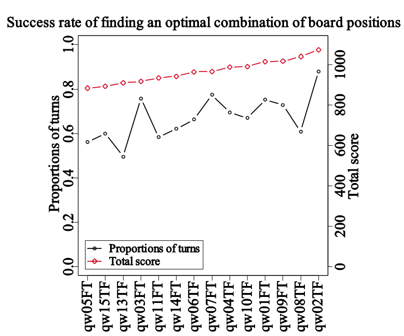

Figure 9: Proportions of turns where subjects were able to find optimal combinations of board positions that lead to the highest possible scores for a chosen combination of tiles.

Discussion

The earlier study involving Qwirkle (Mackey et al., 2011) focused on children of ages 5 to 9. Children in Mackey's study still exhibited significant improvements in general reasoning skills and spatial working memory. The result suggests that the strategy should be simple and intuitive enough to be used by children and yet sophisticated enough to involve cognitive resources ranging from vision to general problem solving. The local maximum strategy suggested by results of our study matches this profile. It is simple but reasonably effective strategy. More importantly, the successful use of this strategy is equally dependent on reasoning skills and visuo-spatial processing skills. This dependency explains why Mackey et al. observed improvement in spatial working memory of subjects.

Our results suggest that the reasoning skills play an important role during the first step of the strategy: identifying the combination of tiles with the highest potential score. However, it remains unclear how subjects decided which combination of tiles to choose. Ideally, it is possible to exhaustively search through all possible combinations of tiles and board positions, the same way the computer opponent does. However, it is highly unlikely that human subjects use exhaustive search due to time and cost inefficiency. It is more likely that subjects employ some form of probabilistic mechanism of making a near-optimal decision under uncertainty (Doya, 2008). Such stochastic mechanism could involve calculating a likelihood of obtaining the highest score given combination of tiles and current board state. Correspondingly, distribution and frequency of color and shape features on the board may affect the likelihood estimation. It was already shown that frequency of both attentively and pre-attentively processed visual features can affect decision making (Nyamsuren & Taatgen, 2013b, 2013c). If it is indeed the case then it will be a direct evidence of visual system directly interfering with reasoning processes. It is also likely that the size of the tile combination plays an important role. More tiles are associated with a higher score. However, bigger tile combination also increases the effort required to find the optimal combination of cells. These and other factors (Busemeyer & Townsend, 1993), such as time pressure, are likely to be considered by subjects in calculating the likelihood of getting highest possible score.

The visuo-spatial processing skills are important during the second step of the strategy. Finding an optimal combination of positions is basically a problem of visual search with multiple targets and multiple partially matching distracters (Anderson, Fincham, Schneider, & Yang, 2012; Hong & Drury, 2002; Horowitz & Wofle, 2001). Targets are the board positions with the highest scores, and distracters are the positions with lower scores. Multiple-target visual search is a demanding process that requires combination of visual feature matching, spatial memory and higher-level mathematical reasoning. In addition to matching visual features, subjects need mathematical reasoning to compare gains across alternative board positions. As such, it seems that visual search is not purely visual and higher-level reasoning is invoked within its context. It is an opposite form of interaction between visual and reasoning systems than the one may be used during the choice of tile combination. Finally, given the complexity of such visual search it is not surprising that subjects fail often during this step. It also explains why subjects are not as good as the computer opponent despite the simplicity of the common strategy. The computer opponent does the perfect visual search.

If we take an overall view of the subjects' strategy in light of the earlier discussion then it is not as simple as having two steps, one with reasoning and one with visual search. Instead, it seems that the visual and reasoning systems are deeply intertwined. On the one hand, reasoning outcome is highly dependent on both attentive and pre-attentive visual knowledge. On the other hand, visual processing requires capability of reasoning on concepts more higher level than visual features.

Conclusion

In this study, we have described the preliminary study of human behavior and strategy in Qwirkle. The next step is to create a cognitive model that can (1) provide empirical validations of the assumptions and hypothesis proposed here, and (2) test whether our explanation is compatible with wider theory of human cognition through the use of ACT-R cognitive architecture (Anderson, 2007).

This study suggests that three types of cognitive resources are most important for this modeling effort. Firstly, declarative memory is necessary to store task specific and general knowledge and rules. Next, the study emphasizes the importance of a visual system as a medium of gathering and processing real-time knowledge. Considering the complexity of the required visual processing the use of the Pre-Attentive and Attentive Vision module (Nyamsuren & Taatgen, 2013a) is required to gain access to such cognitive resources as iconic memory and short-term visual memory. The final type of cognitive resources is fluid reasoning that is capable of integrating declarative and visual knowledge to solve the problem of playing the game. Catell (1987) proposed that fluid reasoning serves as a scaffold that allows us to form and acquire new cognitive skills and knowledge. Halford, Wilson and Phillips (1998) proposed relational integration of fluid reasoning, the ability to jointly consider distinct relationships between stimuli. The Human Reasoning Module (Nyamsuren & Taatgen, 2013d) was developed with the same principle in mind as fluid reasoning. The HRM can serve as a scaffold for deriving new knowledge by combining existing knowledge in the declarative and visual systems.

We have not really touched upon subjects' eye-movement data in this study. The future plans definitely include paying more attention to eye-movement data, especially as a means of estimating the cognitive model's fit to subjects' behavior.

References

Anderson, J. R. (2007). How can human mind occur in the physical universe? New York: Oxford University Press.

Anderson, J. R., Fincham, J. M., Schneider, D. W., & Yang, J. (2012). Using brain imaging to track problem-solving in a complex state space. NeuroImage , 60, 633-643.

Busemeyer, J. R., & Townsend, J. T. (1993). Decision field theory: a dynamic-cognitive approach to decision making in an uncertain environment. Psychological Review , 100 (3), 432-459.

Byrne, R. M., & Johnson-Laird, P. N. (1989). Spatial Reasoning. Journal of Memory and Language , 28, 564-575.

Cattell, R. B. (1987). Intelligence: Its structure, growth and action. Amsterdam: North-Holland.

Doya, K. (2008). Modulators of decision making. Nature Neuroscience , 11 (4), 410-416.

Halford, G. S., Wilson, W. H., & Phillips, S. (1998). Processing capacity defined by relational complexity: implications for comparative, developmental, and cognitive psychology. Behavioral Brain Sciences , 21 (6), 803–831.

Hong, S. K., & Drury, C. G. (2002). Sensitivity and validity of visual search models for multiple targets. Theoretical Issues in Ergonomics Science , 3 (1), 85-110.

Horn, J. L., & Cattell, R. B. (1967). Age differences in fluid and crystallized intelligence. Acta Psychologica (Amsterdam) , 26 (2), 107–129.

Horowitz, T. S., & Wolfe, J. M. (2001). Search for multiple targets: Remember the targets, forget the search. Perception & Psychophysics , 63 (2), 272-285.

Klanderman, D. B., Moore, M. W., Maxwell, M. S., & Robbert, S. K. (2013). Creating Problems and Their Solutions: Service-Learning Through Trinity Mathematics Triathlons, Math Nights, and Math Centers. PRIMUS: Problems, Resources, and Issues in Mathematics Undergraduate Studies , 23 (6), 563-571.

Mackey, A. P., Hill, S. S., Stone, S. I., & Bunge, S. A. (2011). Differential effects of reasoning and speed training in children. Developmental Science , 14 (3), 582-590.

Maloy, R. W., Edwards, S. A., & Anderson, G. (2010). Teaching Math Problem Solving Using a Web-Based Tutoring System, Learning Games, and Students' Writing. Journal of STEM Education: Innovations and Research , 11 (1), 82-90.

Nyamsuren, E., & Taatgen, N. A. (2013a). Pre-attentive and Attentive Vision Module. Cognitive Systems Research , 24, 62-71.

Nyamsuren, E., & Taatgen, N. A. (2013b). Set as instance of a real-world visual-cognitive task. Cognitive Science , 37 (1), 146-175.

Nyamsuren, E., & Taatgen, N. A. (2013c). The synergy of top-down and bottom-up attention in complex task: going beyond saliency models. Proceedings of the 35th Annual Conference of the Cognitive Science Society (pp. 3181-3186). Austin, TX: Cognitive Science Society.

Nyamsuren, E., & Taatgen, N. A. (2013d). From Casual Deduction to Spatial Relations: Bottom-up and Top-down Reasoning Unified. In R. West, & T. Stewart (Ed.), Proceedings of the 12th International Conference on Cognitive Modeling (pp. 203-208). Ottawa: Carleton University.